Public Debt-to-GDP and Velocity of Money Supply

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Abstract

In this paper, we analyse the relationship between the public debt-to-GDP ratio and the velocity of money zero maturity (MZM) in the United States. We conduct Granger causality tests on the two time series over different time samples and subsamples. Our findings suggest that the public debt-to-GDP ratio Granger-causes velocity from 1959Q1 to 2019Q4 with the first lag being significant. Robustness analysis confirm the Granger causality between the two time series. To further investigate these findings and assess the extent to which the level of public debt and fiscal shocks may be determinants of money velocity, we analyse this relationship through the lens of a simple New Keynesian model. We find that the empirical negative relationship can be explained in the model through a change in the public debt driven by a change in government transfers, but not through a change in government spending. Our contribution to literature lies in three aspects. Firstly, we present empirical and theoretical evidence that examining expectations regarding public debt, which we consider highly relevant nowadays. Secondly, our analysis introduces the public debt and the public debt-to-GDP ratio as determinants of money velocity, providing insights into the transmission mechanisms of fiscal policy shocks. Finally, we contribute to the literature by addressing the gap in the literature regarding the prediction of velocity.

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1 Introduction

The velocity of money zero maturity (MZM) in the US has exhibited three different trends since 1959. Firstly, it rose until 1981Q3, then it experienced a steady decline during the recession of 1980 - 1982. After that, it showed a fluctuating trend until 1996Q1, and eventually demonstrated a clear downward trend until 2020. Therefore, as assessed in the literature, the evolution of the money velocity is not constant, although it is often referred to as constant for convenience (Mankiw, 2014).

During the 1980s and 1990s, many a significant part of literature focused on the monetary base, the narrowest measure of money in the economy and the monetary base velocity. In these studies, changes in money velocity are explained as being driven by technological innovation and the creation of substitutes for money (Belongia and Ireland, 2019). Following the approach of Teles and Zhou (2005), Motley (1988), Poole (1991), among others, we focus on the Money Zero Maturity (MZM) aggregate[1]. Teles and Zhou (2005) show that MZM, as a variable for the monetary aggregate, not only represents a more appropriate measure of base money, but is also the money measure necessary to preserve a stable relationship between money supply, the interest rate and GDP.[2] MZM includes all assets that can be easily converted into liquid assets without any maturity constraints, making them readily usable for transactions. As proposed by Ireland (1994), economies experience a shift in monetary technologies as they progress, leading to reduced reliance on currency and increased adoption of advanced forms of exchange, such as electronic currency or time deposits. This suggests that we should focus on the wider definitions of money, and utilising MZM in our analysis offers the advantage of capturing

[1]Money zero maturity includes currency outside the U.S. Treasury, Federal Reserve Banks, and the vaults of depository institutions, demand deposits at commercial banks (excluding those amounts held by depository institutions, the U.S. government, and foreign banks and official institutions) less cash items in the process of collection and Federal Reserve float, other liquid deposits, consisting of other checkable deposits (or OCDs, which comprise negotiable order of withdrawal, or NOW, and automatic transfer service, or ATS, accounts at depository institutions, share draft accounts at credit unions, and demand deposits at thrift institutions) and savings deposits (including money market deposit accounts), savings deposits (which include money market deposit accounts, or MMDAs), balances in retail money market mutual funds (MMMFs) and institutional money funds.

[2]Teles and Zhou (2005) note how the money demand estimations of Lucas Jr (1988) are valid only until the 1980s. The authors show that the reason for this limitation lies in the instability of the measure used for money supply. When using MZM, instead of other monetary aggregates such as M1, the stability of the money demand is restored, even after the 1980s.

financial and technological innovations in the nature of money.

We aim to explore the implications of elevated levels of public debt from an expectations perspective and assess their impact on household spending behaviour. Our investigation revolves around the relationship between these two variables. We start from the hypothesis that changes in public debt-to-GDP ratios and public debt may Granger cause (Granger, 1969) fluctuations in the velocity of money supply. These fluctuations occur primarily through the economic expectations channel. When public debt-to-GDP ratios surge to significant levels, they often imply poor fiscal and financial health within the public sector. This, in turn, has the potential to influence economic expectations and decrease confidence, subsequently affecting inflation and expectations about future taxes, and undermining fiscal credibility. Furthermore, this scenario may give rise to beliefs regarding debt monetisation, as highlighted in Coibion et al. (2021). In their work, drawing insights from a randomised control trial, it becomes evident that information regarding anticipated future high levels in public debt tends to raise households' inflation expectations. Households may also expect some degree of debt monetisation which consequently influences their consumption behaviour. These expectations in theory, should lead to an increase in velocity during the current period, as inflation is expected to manifest in the subsequent period. However, a reduction in velocity may be anticipated in the subsequent period. Hence, an increase in public debt may trigger a decrease in money velocity in the next period. Furthermore, public debt typically sees increases during recessions. Recessions are also characterised by heightened demand for money due to economic uncertainty. As a result, savings tend to rise, leading to a subsequent decrease in money velocity. This preliminary analysis suggests that, employing Granger causality changes in the public debt-to-GDP ratio and public debt may serve as effective predictors of variations in velocity. In this work, we therefore aim to disentangle the main transmission mechanisms of the impact of the public debt-to-GDP ratio on velocity by focusing on the expectations channel.

We focus on the public debt-to-GDP ratio as the increasing trend in the total amount of government debt has become a concern after the Great Financial Crisis. According to the fiscal theory of the price level (Sims (1994), Woodford (1994), Benhabib et al. (2001), Cochrane (2001), Bassetto (2002), Cochrane (2022), Cochrane (2023)) when fiscal policy is active, fiscal policy constitutes one determinant of the price level (thus inflation).

However, this paper limits its analysis to the velocity of money, the public debt-to-GDP ratio, and fiscal policy without investigating the specific link between fiscal policy and inflation.

The second variable of interest is velocity of money supply because, by definition, it measures the rate at which money circulates within the economy, and provides a measure of the economic stance. When the number of transactions in the economy increases, the economy is more likely to expand.[3] Reversely, when the economy grows, money demand increases which causes a rising money-to-GDP ratio and a fall in velocity. This has been researched empirically in previous literature (see for example Friedman (1959) and Ireland (1991)) and recently formalised by Mele and Stefanski (2019).[4] Moreover, following the seminal work of Lucas Jr and Stokey (1987) among others, money velocity is recognised as a determinant of inflation, influencing it through the expectations channel. While many studies have analysed the relationship between velocity and inflation, our specific focus is once more not on inflation, but rather on exploring the link between velocity and fiscal policy.

The gap in literature that we aim to fill is related to the predictability of velocity. If velocity were stable or constant, forecasting would be easier, providing a credible instrument to measure economic growth. However, velocity is not constant and its predictability is not straightforward. The ability to forecast velocity can be particularly insightful, especially, though not exclusively, during economic and financial crises (Anderson et al., 2017). Therefore, this work aims to add a step to the process of forecasting velocity, by providing the public debt as one possible determinant of it. The Granger causality analysis between the public debt-to-GDP ratio and money velocity is suitable for this purpose.

To construct a measure for velocity, Gordon et al. (1998) use real money supply, real consumption and investments. Following Isard and Rojas-Suarez (1986), Piazzesi et al. (2019) and the FRED database construction of velocity, we use nominal variables for the empirical part of our work, because nominal variables provide a comprehensive view of

[3] See https://www.stlouisfed.org/on-the-economy/2014/september/what-does-money-velocity-tell-us-about-low-inflation-in-the-us

[4] Mele and Stefanski (2019) focus on industrialisation to measure economic growth. While they employ a two-sector model to explain the relationship between GDP growth and velocity, we focus on the debt-to-GDP ratio and velocity in a small New Keynesian model with households, firms, a government and a central bank.

the transactions in the economy. Firstly, we present the results obtained from Granger causality tests that we ran between the US time series of velocity of MZM and the public debt-to-GDP ratio. However, results are robust when considering real variables as well. We consider data from 1959Q1 to 2019Q4, that is the longest period over which data for these two time series can be retrieved from the FRED Economic Data database from the St. Louis FED.[5]

We find strong evidence against the hypothesis that velocity is not granger-caused by the public debt-to-GDP ratio, mainly for the entire sample, and between 1959Q1 and 1995Q4. We choose to split our samples based on a graphical analysis of the evolution of the two time series. The second plot of figure 1 shows that the percentage change of public debt-to-GDP and money velocity follow opposite directions until right after 1995, following a countercyclical pattern. However, from approximately 1996Q1 on, the debt-to-GDP ratio and the money velocity exhibit a similar path, with a brief exception, right before and after the year 2000. Therefore, to determine the length of the first time subsample, we start from 1959Q1 until 1994Q1 and add one quarter at a time. We then run a Granger causality test on each of these subsamples. We find out that until 1995Q4, there is still a strong Granger causality relationship between the two time series. From 1996Q1 until 2019Q4 we again obtain evidence of public debt being a granger-cause for money velocity, but as expected, the coefficient significance is lower. To validate our findings, robustness analyses are performed. One of the robustness analysis, which are presented in section 6, is performed after removing the period related to the Great Financial Crisis from the data. We obtain that the initial results for the entire sample are confirmed.

To explain the dynamics of velocity and the role of public debt, we present a small New Keynesian model built upon the framework developed by Galí (2020). Our model incorporates a fiscal block that encompasses government spending and transfers. We model the increase in public debt as either an increase in government spending or an increase in government transfers. Utilising the linearity of the model, we explore the effects of a fiscal shock on velocity. Our model is capable of generating a negative relationship between public debt-to-GDP and money velocity when a transfers shock is implemented. An increase in government spending induces an expansionary effect on output consequently

[5] https://fred.stlouisfed.org

leading to an increase in velocity.

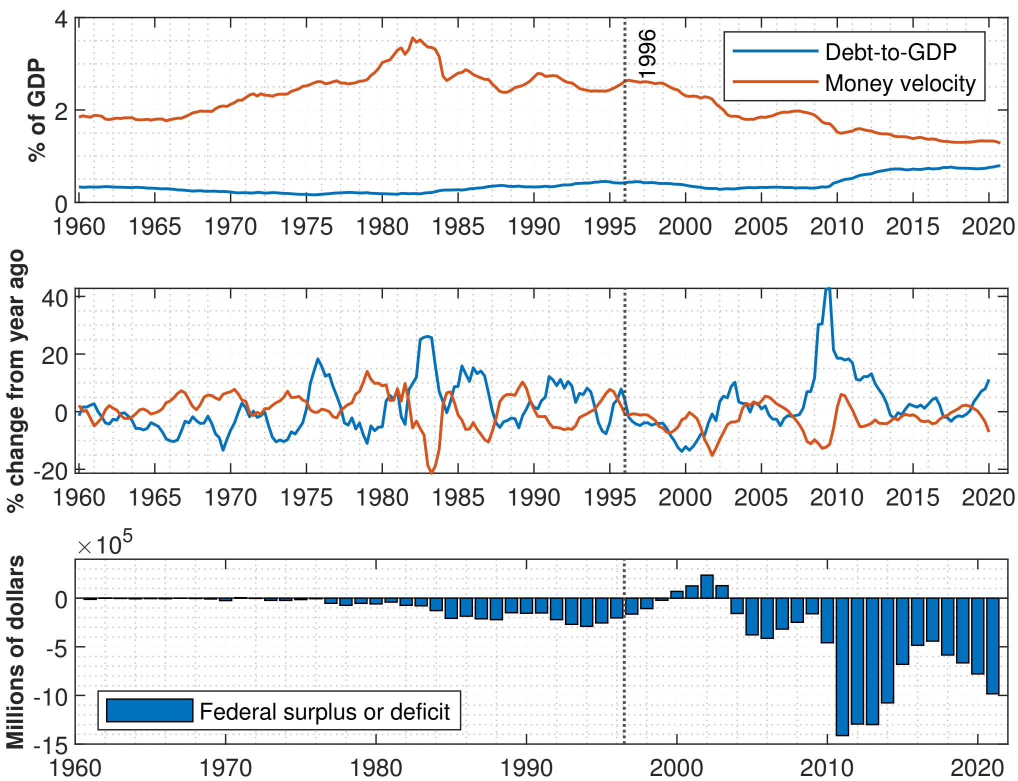
This paper is organised as follows. Section 2 briefly describes the literature on expectations about fiscal policy. Section 3 shows the empirical evidence of the relationship between the public debt-to-GDP and the velocity of money supply. Section 4 describes the theoretical model. Section 5 discusses the model impulse response functions. Section 6 describes robustness checks and section 7 concludes.

2 Brief literature review on fiscal expectations

Several authors have investigated topics related to fiscal expectations in the literature (Calvo, 1988; Gordon et al., 1998; Leeper, 2009; Bernasconi et al., 2009). Bernasconi et al. (2009) conducted a laboratory experiment to assess the extent to which fiscal variables affect fiscal expectations. The authors ran the experiment in a controlled environment using real economic data, which the participants were shown and understood through adaptive expectations. The authors found that expectations are affected by the data. However, their work primarily focuses on expectations about fiscal variables through an experimental analysis. Calvo (1988) discusses the credibility of the government and the expectations about future repayment of the public debt. The focus of the analysis is on the non-uniqueness of equilibria, which can arise from the existence of government bonds and tax postponement. More recently, Coibion et al. (2021) use a Randomised Control Trial approach to assess inflation and fiscal expectations of households.

Leeper (2009) addresses the anchoring of fiscal expectations and the differences between monetary policy expectations and fiscal policy expectations. The author emphasises the effects of anticipated tax changes and the transparency of government actions. Considering the impact of the Great Financial Crisis and the growing emphasis on the fiscal aspect, Leeper (2009) argues that expectations about fiscal policy should be addressed similarly to expectations about monetary policy, due to their relevance in achieving macroeconomic stability. Our focus is on the determinants of the velocity related to fiscal policy. To the best of our knowledge, not much work has been done on the relation between the velocity of money supply, fiscal policy and public debt. Our paper shares conceptual similarities with the work of Gordon et al. (1998). The authors focus on the general equilibrium determinants of velocity. The paper explores the trends in the velocity of base money in the US from 1960 to 1997 and whether these trends can be

Figure 1: Debt-to-GDP, MZM velocity and surpluses/deficits in the US



Note: The first plot illustrates the relationship between the public debt-to-GDP ratio and the MZM velocity. The second plot shows the percentage change of both variables compared to the previous year. The histogram provides an overview of the evolution of deficits and surpluses in the United States.

explained by endogenous responses to changing expectations about monetary and fiscal policy. The authors use a model that maps policy expectations into portfolio decisions, making equilibrium velocity a function of expected future money growth, tax rates, and government spending. They find that the observed secular movements in velocity can be accounted for exclusively by endogenous responses to policy expectations. While our empirical results align with those obtained by Gordon et al. (1998), our study differs in that we provide empirical evidence of Granger causality between public debt-to-GDP ratio and velocity over an extended sample period. Furthermore, we analyse the evidence of these empirical findings within the framework of a New Keynesian model. A dynamic stochastic general equilibrium framework helps explain different transmission mechanisms of two different fiscal policy shocks to money velocity.

3 Empirical evidence

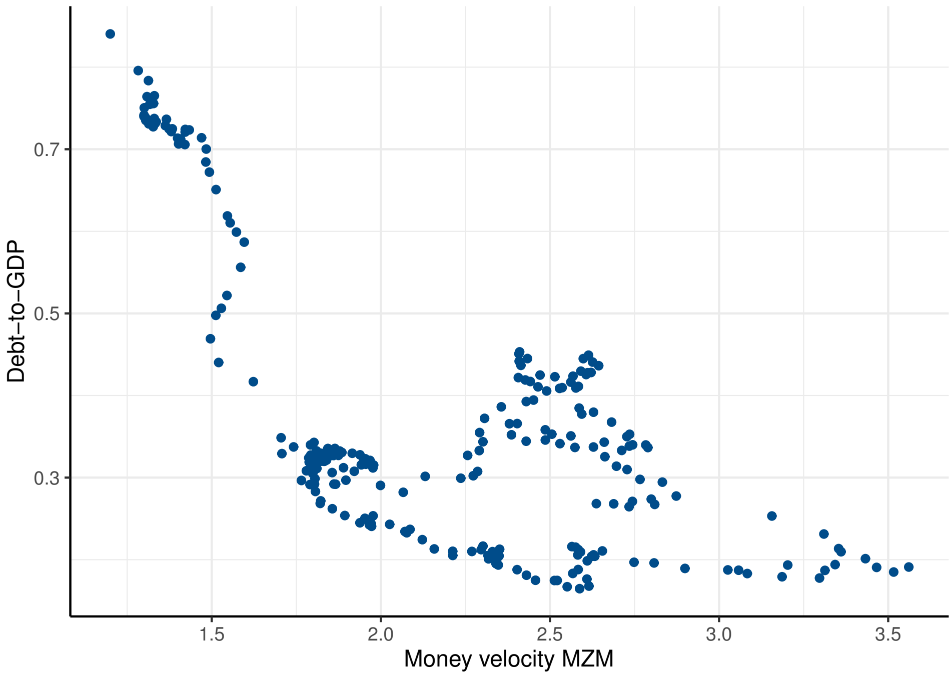
We evaluate several US time series representing the public debt-to-GDP ratio and MZM velocity. The money velocity is constructed as the ratio between nominal GDP and the MZM money stock. The MZM velocity is given by $\frac{P_t Y_t}{M_t}$, where Y_t is nominal GDP, P_t is the price level and M_t represents the monetary stock. Similarly, the public debt-to-GDP ratio is given by $\frac{B_t}{P_t Y_t}$, where B_t is the public debt. To construct the public debt-to-GDP ratio, we follow Bianchi et al. (2023) and others, that use the market value of marketable treasury debt. This approach provides a more reliable measure for quantifying the debt burden of the United States. All the time series used in the analysis are retrieved from the Federal Reserve Economic Data database of the Federal Reserve Bank of St. Louis and are detailed in the Appendix 9.

Figure 2 illustrates the relationship between quarterly raw data for public debt-to-GDP and money velocity over the period 1959Q1 - 2019Q4. The second plot of the figure reveals a negative relation between the two variables. A correlation test conducted on the two stationary time series yields a Pearson correlation coefficient of -0.3788.

3.1 Empirical results

To test for Granger causality (Granger, 1969), we perform a regression of money velocity on its own lagged values as well as on lagged values of the public debt-to-GDP ratio. The null hypothesis tested is that the estimated coefficients corresponding to the lagged values of the debt-to-GDP ratio are equal zero. Rejection of the null hypothesis is equivalent to rejecting the hypothesis that the public debt-to-GDP does not granger-cause the MZM velocity. Furthermore, considering that no specific pattern is assumed in the Granger causality analysis, we regress the public debt-to-GDP ratio on its own lagged values and on the lagged values of the velocity. Rejection of the null hypothesis in this case indicates rejection of non-granger causality in the opposite direction: from money velocity to the public debt-to-GDP ratio.

Figure 2: Debt-to-GDP and MZM velocity



Note: The figure represents the relationship between the two time series over the time sample 1959 - 2019. On the x-axis, money velocity is represented by $GDP/MZMNS$. On the y-axis, Debt-to-GDP is represented as $MVMTD027MNFRBDAL/GDP$. Source: FRED, Federal Reserve Bank of St. Louis.

We use a VAR(p) model of the type:

$$V_t = \phi + \sum_{i=1}^{p} \phi_i^V V_{t-i} + \sum_{i=1}^{p} \phi_i^B B_{t-i} + \epsilon_t^V$$
$$B_t = \gamma + \sum_{i=1}^{p} \gamma_i^V B_{t-i} + \sum_{i=1}^{p} \gamma_i^B V_{t-i} + \epsilon_t^B$$

where V_t is the MZM velocity and B_t represents the public debt-to-GDP ratio. The number of optimal lags in the model is chosen based on the Akaike Information Criterion (AIC) (Akaike, 1974) and on the Bayes' Criterion (BIC) (Schwarz, 1978).

Table 1 shows the results of Granger causality tests between the variables for the entire sample 1959Q1 until 2019Q4, as well as for the two subsamples. The first part of the table reports for each time sample the coefficient estimate of each lag. Results for both information criteria are displayed. Although the lag length changes from 1959 until 2019 and from 1959 until 1996 based on the information criterion used, coefficients estimates at the first lag remain very similar for both AIC and BIC. It is also worth noting that over the whole sample, the debt coefficient at the fifth lag results to be significant and positive. The second part of the table reports the lag length chosen based on the information criterion and the adjusted R-squared of the regression. It is important to note that in each regression are included both the lags of the dependent variable, as well as the lags of the independent variable. The third part of the table displays the F-statistics and the p-value associated with the Granger causality test.

Table 2 shows the same results, but when testing for the inverse causality, from the money velocity to the public debt-to-GDP. Interestingly, here none of the analyses yields significant coefficients for velocity, suggesting that the null hypothesis of non-Granger causality in the direction money velocity to public debt-to-GDP cannot be rejected.

3.2 Discussion of empirical results

An increase in public debt and the public debt-to-GDP ratio could lead to expectations of further future public debt increases. These expectations of deteriorating public finances can trigger an increase in inflation expectations, as demonstrated in Coibion et al. (2021). Consequently, this can lead to reduced consumption and output in the following period. As a result, velocity decreases in the next period. Conversely, a decrease in public debt and the public debt-to-GDP ratio may predict an increase in velocity in the next period through lower inflation expectations. Furthermore, before the Volcker era, due to the Fed's monetary mandate, households may have anticipated some degree of future debt monetisation. However, post-Volcker, this possibility disappeared as the Fed adopted an inflation targeting mandate and enhanced its credibility, acting as an anchor for inflation expectations. Even though analysis run on subsamples based on the Volcker period do not yield significant changes in the coefficients, the break in 1996 indicates that the Granger causality results became weaker from that point onwards. Nonetheless, the negative relationship still persists even after the break. From 1996Q1 to 2019Q4 our results once again indicate a negative relationship between the public debt-to-GDP ratio and MZM velocity at the first lag. It is worth noting that this time period encompasses the Global Financial Crisis, which coincided with substantial changes in the US public debt and in the MZM money supply. These results suggest that the Great Financial Crisis (GFC) may have not affected the significance of the regression coefficient between the two indicators. However, with respect to the entire time period 1959Q1-2019Q4, from

1996Q1 until 2019Q4 the coefficient is significantly lower, and its estimate is one half of the coefficient estimate over the period 1959Q1-1995Q4.

The period starting from 1996 on coincides with Bill Clinton's presidency second term, characterised by the largest budget surpluses in the US government history, and the first fiscal surpluses the US experienced since the 1960s. The third plot in figure 1 shows the evolution of federal deficits and surpluses throughout the analysed time period.

In 2003 the war in Iraq began, leading to an escalation in the fiscal burden and a transition from surpluses to deficits. In 2008 the GFC begun and during the period onwards monetary policy underwent changes that included unconventional measures aimed at managing the effects of the recession. One such measure was Quantitative Easing (QE), through which the Federal Reserve acquired substantial amounts of long-term treasury debt. Given the objectives of QE, these large-scale purchases had an impact on both US money supply and the total federal debt. These policies may have had an impact on the relationship between the public debt and the public debt-to-GDP, and velocity of money supply. Results for robustness analysis excluding the GFC from the sample are presented in tables 4 and 5.

We present a small theoretical model to enhance our understanding of the transmission mechanisms through which fiscal policy affects the velocity of money supply. The model is described in the next section.

4 Theoretical model

In this section we introduce the theoretical model. The model follows the New Keynesian framework in Galí (2020). Households consume, work and hold money. Firms are of two types: a final goods firm and an intermediate goods firm. Final good firms produce the homogeneous final product by aggregating intermediate goods. This final product is then sold to households on a perfectly competitive market. Intermediate good firms face price rigidities, adjusting their prices following Calvo (1983). Government follows a budget constraint including bonds, government spending and government transfers.

4.1 Households

The economy is populated by a continuum of households. The households consume, supply labour force to firms, and own and receive dividends from those firms. They also hold one-period riskless bonds. The implicit form of the household's utility is the following:

$$E_0 \sum_{t=0}^{\infty} \beta^t \mathcal{U}\left(C_t, L_t, N_t\right)$$

with period $\mathcal{U}(.)$ utility function taking the form: $\mathcal{U} = \left(U\left(C, L\right) - V\left(N\right)\right)$ where $L_t \equiv M_t/P_t$ are the real money balances. M_t is the nominal stock of money and P_t is the price level, C_t is consumption, N_t is employment. As such, the utility function assumes non-separability between consumption and money balances. However, for simplicity, we have chosen to calibrate the household's first-order condition in a manner that implies separability between consumption and real balances.

The household's budget constraint is standard and writes:

$$P_t C_t + B_t + M_t = B_{t-1}\left(1 - i_{t-1}\right) + M_{t-1} + W_t N_t + D_t + P_t T_t$$

where B_t is a one-period riskless bond, W_t is the nominal wage, N_t represents the hours worked. As owners of the firms, D_t are dividends received. T_t represent the government transfers. Given the discount factor β, the households maximise their utility function.

4.2 Firms

There are two types of firms in the economy: final good firms and intermediate goods firms. Final good firms pack intermediate goods into a final good, that is then sold to households in a perfectly competitive market. Intermediate good firms hire labour from households, that also own the intermediate good firms, and produce intermediate goods that are then sold to the final good firms.

Intermediate goods are produced with the following technology:

$$Y_{it} = N_{it}^{1-\alpha}$$

where N_t represents the labour. The real marginal cost MC_t^r is:

$$MC_t^r = W_t P_t^{-1}\left(1-\alpha\right)^{-1} N_t^{\alpha}$$

Table 1: Public debt-to-GDP → MZM velocity

	AIC			BIC		
	1959Q1 - 2019Q4	**1959Q1 - 1995Q4**	**1996Q1 - 2019Q4**	**1959Q1 - 2019Q4**	**1959Q1 - 1995Q4**	**1996Q1 - 2019Q4**
Debt(-1)	**-0.25*** **	**-0.30*** **	**-0.15** **	**-0.23*** **	**-0.32*** **	**-0.15** **
	(0.04)	(0.08)	(0.05)	(0.05)	(0.08)	(0.05)
Debt(-2)	0.06	-0.06	-0.01	0.08	-	-0.01
	(0.05)	(0.09)	(0.08)	(0.06)	(-)	(0.08)
Debt(-3)	0.02	-0.09	-	0.03	-	-
	(0.06)	(0.09)	(-)	(0.06)	(-)	(-)
Debt(-4)	-0.02	-0.10	-	-	-	-
	(0.06)	(0.07)	(-)	(-)	(-)	(-)
Debt(-5)	**0.12***	-	-	-	-	-
	(0.05)	(-)	(-)	(-)	(-)	(-)
Lag length	5	4	2	3	1	2
R^2	0.36	0.35	0.28	0.33	0.32	0.28
GC: F-test	8.5843	10.907	5.6393	10.792	34.954	5.6393
GC: p-value	**(8.735e-08)**	**(3.269e-08)**	**(0.00)**	**(7.211e-07)**	**(9.537e-09)**	**(0.00)**

*Note: The table reports results for regressions and the Granger causality test with H0: Public debt-to-GDP does not granger-cause money velocity. Standard errors of estimates are reported in parentheses. "Lag length" reports the number of lags in the model according to the AIC and BIC. GC: F-test and GC: p-value refer to the F-statistics and to the p-value of the Granger causality analysis, respectively. ***, **, and * denote rejection of the HO at the 1%, 5% and 10% significance level, respectively.*

Table 2: MZM velocity → public debt-to-GDP

	AIC			BIC		
	1959Q1 - 2019Q4	**1959Q1 - 1995Q4**	**1996Q1 - 2019Q4**	**1959Q1 - 2019Q4**	**1959Q1 - 1995Q4**	**1996Q1 - 2019Q4**
Velocity(-1)	0.11	0.15	0.19	0.12	0.05	0.19
	(0.12)	(0.13)	(0.32)	(0.11)	(0.10)	(0.32)
Velocity(-2)	-0.03	0.06	0.24	-0.01	-	0.24
	(0.17)	(0.18)	(0.32)	(0.16)	(-)	(0.32)
Velocity(-3)	0.23	0.34	-	0.06	-	-
	(0.17)	(0.20)	(-)	(0.16)	(-)	(-)
Velocity(-4)	-0.18	-0.02	-	-	-	-
	(0.13)	(0.11)	(-)	(-)	(-)	(-)
Velocity(-5)	-0.05	-	-	-	-	-
	(0.17)	(-)	(-)	(-)	(-)	(-)
Lags	5	4	2	3	1	2
R^2	0.26	0.23	0.30	0.25	0.12	0.30
GC: F-test	1.2517	2.1374	0.88804	0.59972	0.20325	0.88804
GC: p-value	(0.28)	(0.08)	(0.41)	(0.62)	(0.65)	(0.41)

*Note: The table reports results for the Granger causality test with H0: Money velocity does not Granger-cause public debt-to-GDP. Standard errors of estimates are reported in parentheses. "Lag length" reports the number of lags in the model according to the AIC and BIC. GC: F-test and GC: p-value refer to the F-statistics and to the p-value of the Granger causality analysis respectively. ***, **, and * denote rejection of the HO at the 1%, 5% and 10% significance level, respectively.*

Intermediate firms set their prices following Calvo (1983). A fraction of firms equal to $1-\theta$ can change their price, while the remaining fraction θ have to keep their prices unchanged. For this reason, their FOC is:

$$E_t \sum_{k=0}^{\infty} \beta^k \theta^k Q_{t,t+k} Y_{it+k|t} \left(P_t^* - \mu_{t+k} \, MC_{t+k|t}^n \right) = 0$$

where $Q_{t,t+k}$ is the nominal discount factor for firms and the discount factor for the households, $Y_{it+k|t}$ is output produced in period $t+k$ given the price set in period t, P_t^* is the optimal price, μ_{t+k} is the price mark-up and $MC_{t+k|t}^n$ is the nominal marginal cost. The final good firms produce their goods with the following technology:

$$Y_t = \left(\int_0^1 Y_{it}^{\frac{1}{1+\epsilon}} \, di \right)^{1+\epsilon}$$

4.3 Monetary policy

The central bank sets the nominal interest rate based on the level of inflation and output in the economy. The standard Taylor rule reads as follows:

$$R_t = \Pi_t^{\phi_\pi} Y_t^{\phi_y} \tag{1}$$

where R_t is the gross nominal interest rate, and ϕ_π and ϕ_y are the weights of the interest rate on inflation and output respectively.

4.4 Fiscal policy

The non-maximising government follows a budget constraint

$$P_t G_t + B_{t-1} \left(1 + i_{t-1}\right) = B_t - P_t T_t + \left(M_t - M_{t-1}\right) \tag{2}$$

where G_t is the government spending, T_t are the transfers to households and $(M_t - M_{t-1})$ represents the money growth. It is worth noting that instead of the government transfers, we could have alternatively used lump-sum taxes with the opposite sign, and the results would remain unchanged. Public debt increases are generated by an increase in government spending and/or an increase in government transfers. Moreover, government transfers and government spending follow fiscal rules that link their evolution to the evolution of debt in the previous period. The fiscal rule for transfers is:

$$T_t = B_{t-1}^{\psi_{bt}{}^{-1}} t_t^* \tag{3}$$

and for government spending:

$$G_t = B_{t-1}^{\psi_{bg}-1}\, g_t^* \tag{4}$$

where ψ_{bt} and ψ_{bg}, together with the Taylor rule parameter ϕ_π are calibrated to ensure a regime with active monetary policy and passive fiscal policy. The exogenous components $\hat{t}_t^*$ and $\hat{g}_t^*$ describe shocks processes for transfers and for government spending respectively and are represented by the following linearised AR(1) processes:

$$\hat{g}_t^* = \delta_g\, \hat{g}_{t-1}^* + \epsilon_t^g \tag{5}$$

$$\hat{t}_t^* = \delta_t\, \hat{t}_{t-1}^* + \epsilon_t^t \tag{6}$$

where the stochastic components ϵ_g and ϵ_t represent the fiscal shocks.

4.5 Velocity of money supply

We adopt the definition of velocity as employed in the Federal Reserve Bank of St. Louis Economic Data database (FRED), where it is defined as the ratio of nominal GDP to the monetary aggregate.

$$V_t \equiv \frac{P_t Y_t}{M_t} \tag{7}$$

As stated in the first section, empirical data indicate that velocity exhibits variability. Therefore, we treat velocity as a variable and we include the aforementioned identity in our general equilibrium framework. In this way, we can analyse the separate influences that GDP and money supply exert on it.

4.6 Equilibrium

The equilibrium in the economy is given by:

$$Y_t = C_t + G_t \tag{8}$$

To review the model equilibrium conditions and the full set of linearised equations, please refer to the Appendix 7.

4.7 Calibration

The calibration follows previous related literature and is displayed in table 3. $\beta = 0.995$ implies a 2% annual real interest rate. To calibrate price rigidities, we follow a standard

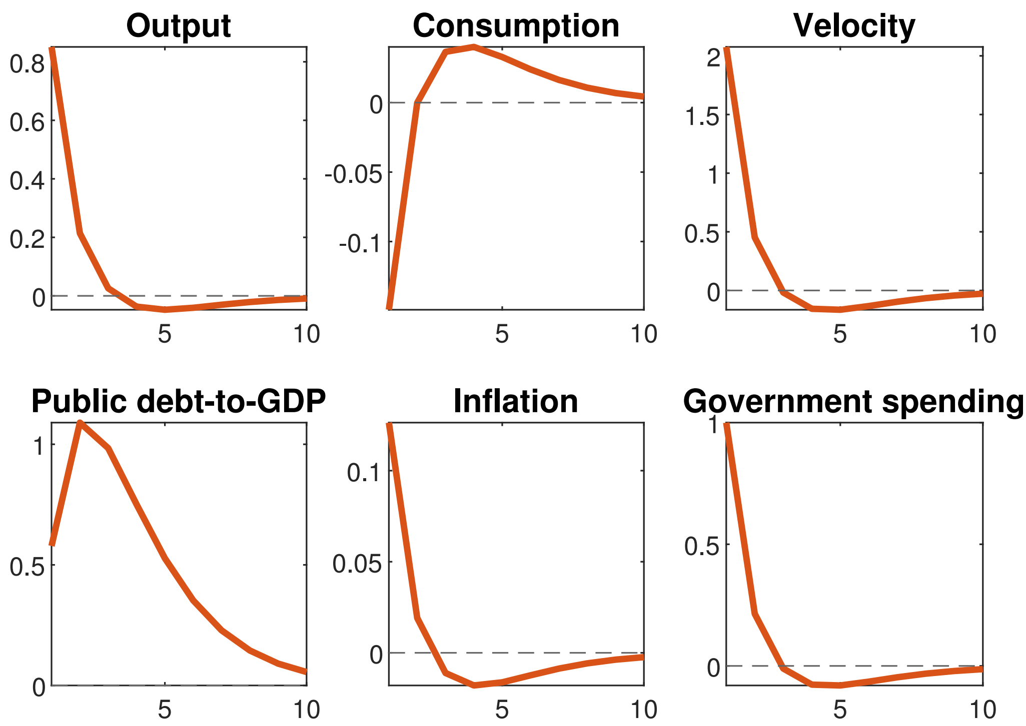
Table 3: Calibrated parameters and source

Parameter	Value	Source
β Household's discount factor	0.995	Galí (2015, 2020)
σ Inverse elasticity of substitution	1	Galí (2015)
φ Inverse Frisch elasticity of labour supply	1	Davig and Leeper (2011)
α labour share in Cobb Douglas function	0.25	Galí (2015, 2020)
θ Calvo parameter	0.75	Galí (2015, 2020)
η semi-elasticity of money demand to interest rate	7	Galí (2015, 2020)
ϵ Elasticity of substitution in CES utility	9	Galí (2015, 2020)
ν CRRA coefficient for real balances	0	Galí (2020)
ϕ_π Interest rate response to inflation	1.5	Taylor (1999)
ϕ_yInterest rate response to output	0.125	Galí (2015)
δ_t AR persistence parameter transfers	0.5	Galí (2020)
δ_g AR persistence parameter government spending	0.5	Galí (2020)
ψ_{bg} Government spending response to debt	0.2	Leeper et al. (2017)
ψ_{bt} Government transfers response to debt	0.2	Leeper et al. (2017)
$\bar{b}$ Steady state value of government debt	0.37	Mean value our sample
$\bar{g}$ Steady state value of government spending	0.19	Mean value our sample
χ Steady state inverse velocity of money supply	0.42	Mean value our sample

value in literature, $\theta = 0.75$, that implies a price change every $\frac{1}{1-\theta} = 4$ quarters. The results are robust for different price rigidities calibrations. We set $\nu = 0$ that introduces separability between consumption and real balances in the households' utility function.[6] We calibrate the parameters for the government transfers and government spending response to debt, ψ_{bt} and ψ_{bg}, as in Leeper et al. (2017). Again, the impulse response functions of fiscal shocks are robust to different specifications of these parameters. The steady state government public debt-to-GDP is set to 37%, which is the average value in our sample. We also calibrate steady state government spending based on the average value in our sample. Following Leeper et al. (2010) among others, we model steady state government transfers as a residual of the government budget constraint, after having defined $\bar{g}$ and $\bar{b}$. Finally, we retrieve the steady state inverse velocity of money supply from our data sample, and corresponds to a velocity average value of 2.38.

[6]For the related equations, please see the appendix section 8.D.3

Figure 3: 1% government spending increase

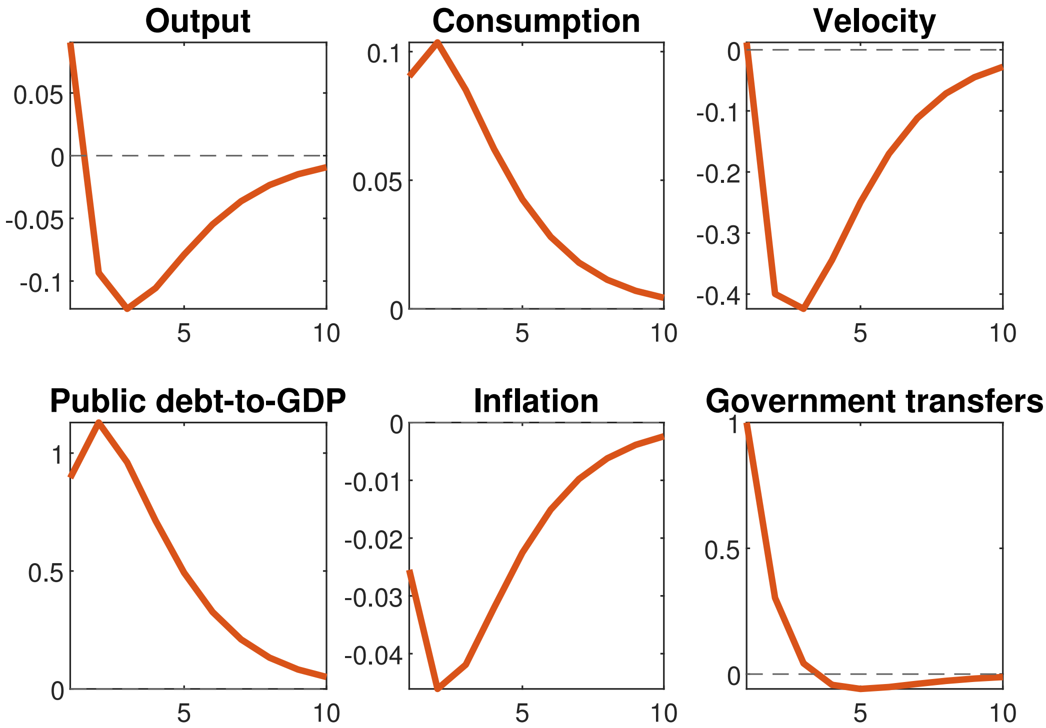


5 Fiscal policy shocks and velocity

Figure 3 shows impulse response functions for a one percent increase in government spending. Figure 4 shows impulse response functions for a one percent increase in transfers employing the baseline calibration showed in table 3.

In this New Keynesian framework, a one percent increase in government spending has a positive impact on output that lasts for two quarters. As a result, inflation raises for approximately two quarters and the central bank react increasing the interest rate. This has a negative wealth effect on consumption at least in the first quarter. Public debt increases, and in our model, the public debt-to-GDP ratio also increases. This is a standard result obtained in a New Keynesian model following a positive government spending shock. Interestingly, the impact on velocity is positive for at least two quarters. This finding appears to contrast with the empirical results discussed in Section 3, where an increase in public debt leads to a contrary evolution of velocity. However, considering the definition of money velocity $V_t \equiv \frac{P_t Y_t}{M_t}$, the magnitude of the denominator and the numerator are important factors that drive the response of velocity. When output

Figure 4: 1% government transfers increase



increases more than money supply, velocity increases. In our model, this is the case, as an increase in government spending exerts an expansionary impact on the economy. Moreover, the central bank controls the nominal interest rate as a primary monetary policy tool and money supply is adjusted consequently. As a result of a decrease in the real rate, consumption is crowded out. A decrease in consumption and an increase in the nominal interest rate lead to a decrease in money demand and, consequently, a decrease in money supply. All these mechanisms imply that output increases more than money supply, which causes velocity to respond positively. If, on the contrary, the money supply would increase more than output, this would have a dampening effect on velocity.

The case of a one percent increase in government transfers produces different effects on the economy and on velocity. Following an increase in government transfers, households' perception of their own wealth increases. This triggers an increase in consumption, that stays above the equilibrium level at least for ten quarters. Output experiences only an increase on impact, just to fall abruptedly under the equilibrium starting from the second period on. This is the consequence of the financing method for an increase in government transfers, which implies a decrease in government spending. Driven by the fall in output, hours worked experience only a short-lived increase just to decrease soon after. Real wages increase on impact and experience a decrease from the second period on. Money supply adjusts upwards to the level of money demand, that increases, driven by the increase in the aggregate demand and in consumption. Finally, velocity experiences only a very brief increase above equilibrium, and subsequently stays negative for at least ten quarters. Therefore, in this case, money supply increases more than output, which is the driver of an decrease in money velocity. It is worth noting that the standard Ricardian equivalence case does not hold in this setting, as we use two fiscal rules, one for transfers and one for the government spending.

In the two exercises above, the factors influencing a change in velocity are the magnitudes of the changes in output and money supply. An increase in government spending not only produces an expansionary effect from an economic growth perspective but also increases velocity. Conversely, in the case of transfers, output does not expand more than the increase in money supply required by the heightened aggregate demand following the increase in transfers.

6 Robustness analysis

6.1 Exclusion of the Great Financial Crisis period

Tables 4 and 5 present robustness checks on the entire sample, excluding the period of the Great Financial Crisis (GFC). We remove the period 2008Q1 - 2009Q2, which is recognised as a recessionary period following the FRED database.

Firstly, the results indicate no change in the lag length and significance with respect to the main analysis concerning the AIC. Secondly, the coefficient of the debt-to-GDP first lag, the standard error and the p-value are very similar to the ones obtained including the Great Financial Crisis (GFC) in the sample. This is an interesting result, suggesting that the GFC did not have a major significant impact on the Granger causality relationship between money velocity and public debt-to-GDP. Concerning the second criterion, BIC there are slight changes with respect to the main analysis, of which the biggest one is a reduced significance level, reflected by the rejection of the H0 at the 5% significance level.[7]. The coefficient however is still negative, significant and the Granger causality test

[7] See table 1 and table 2 for a comparison.

Table 4: Public debt-to-GDP → MZM velocity excluding the GFC

	AIC	BIC
	1959Q1 - 2019Q4	**1959Q1 - 2019Q4**
Debt(-1)	**-0.23*** **	**-0.20** **
	(0.06)	(0.07)
Debt(-2)	0.03	-
	(0.07)	(-)
Debt(-3)	0.02	-
	(0.06)	(-)
Debt(-4)	-0.08	-
	(0.06)	(-)
Debt(-5)	**0.13***	-
	(0.06)	(-)
Lag length	5	1
R^2	0.32	0.28
GC: F-test	6.8214	22.595
GC: p-value	**3.814e-06**	**2.666e-06**

*Note: The table reports results for the Granger causality test with H0: Public debt-to-GDP does not Granger cause money velocity. Standard errors of estimates are reported in parentheses. "Lag length" row reports the number of lags in the model according to AIC and BIC. GC: F-test and GC: p-value refer to the F-statistics and to the p-value of the Granger causality analysis respectively. ***, **, and * denote rejection of the H0 at the 1%, 5% and 10% significance level, respectively.*

still indicates a negative Granger causality relationship between public debt-to-GDP and MZM velocity.

Finally, 5 confirm no reverse Granger causality relationship, from velocity to the public debt-to-GDP.

6.2 Public debt and money supply

A further set of robustness checks is to perform Granger causality analyses between the public debt and the MZM money stock. The detailed description of the series used can be found in Appendix 9. In the main analysis, we found a significant and negative estimate between public debt-to-GDP and money velocity. Therefore, concerning the hypothesis of Granger causality between public debt and money supply, in a regression of money supply on its lagged values and lagged values of public debt, we would expect a significant

Table 5: MZM velocity → public debt-to-GDP excluding the GFC

	AIC	BIC
	1959Q1 - 2019Q4	**1959Q1 - 2019Q4**
Velocity(-1)	0.09	0.10
	(0.11)	(0.10)
Velocity(-2)	0.00	-
	(0.17)	(-)
Velocity(-3)	0.23	-
	(0.18)	(-)
Velocity(-4)	-0.15	-
	(0.11)	(-)
Velocity(-5)	-0.02	-
	(0.16)	(-)
Lag length	5	1
R^2	0.32	0.15
GC: F-test	1.4873	1.256
GC: p-value	0.19	0.26

*Note: The table reports results for the Granger causality test with H0: Money velocity does not Granger-cause the public debt-to-GDP. Standard errors of estimates are reported in parentheses. "Lag length" row reports the number of lags in the model according to AIC and BIC. GC: F-test and GC: p-value refer to the F-statistics and to the p-value of the Granger causality analysis respectively. ***, **, and * denote rejection of the H0 at the 1%, 5% and 10% significance level, respectively.*

Table 6: Public debt → MZM money stock

	AIC	BIC
	1959Q1 - 2019Q4	**1959Q1 - 2019Q4**
Debt(-1)	**0.31*** **	**0.29*** **
	(0.06)	(0.06)
Debt(-2)	**0.22** **	0.11
	(0.09)	(0.07)
Debt(-3)	**0.24** **	0.10
	(0.09)	(0.05)
Debt(-4)	**0.19** **	-
	(0.06)	(-)
Debt(-5)	0.12	-
	(0.11)	(-)
Lag length	5	3
R^2	0.36	0.33
GC: F-test	13.157	17.476
GC: p-value	**(5.579e-12)**	**(9.095e-11)**

*Note: The table reports results for the Granger causality test with H0: The public debt does not Granger-cause money stock MZM. Standard errors of estimates are reported in parentheses. "Lag length" row reports the number of lags in the model according to AIC and BIC. GC: F-test and GC: p-value refer to the F-statistics and to the p-value of the Granger causality analysis respectively. ***, **, and * denote rejection of the H0 at the 1%, 5% and 10% significance level, respectively.*

and positive estimate at least for the first lag coefficient of public debt.

Additionally, when testing for the Granger causality in the inverse direction, from money supply to public debt, the coefficients should not be significant. Figures 6 and 7 show the results. Firstly, the coefficient estimates of the first lag of the independent variable are still significant and very close to the coefficient estimates in the main analysis for both AIC and BIC. Their sign is now reversed, as expected. As shown in figure 6, concerning the AIC, the second, third and fourth lag coefficients are significant and positive as well. Finally, results in table 7 confirm that the null hypothesis of non-Granger causality between money supply and public debt cannot be rejected.

Table 7: MZM money stock → public debt

	AIC	BIC
	1959Q1 - 2019Q4	**1959Q1 - 2019Q4**
Velocity(-1)	-0.001	0.02
	(0.14)	(0.12)
Velocity(-2)	-0.12	-0.09
	(0.18)	(0.14)
Velocity(-3)	-0.28	-0.24
	(0.19)	(0.14)
Velocity(-4)	-0.03	-
	(0.22)	(-)
Velocity(-5)	0.01	-
	(0.17)	(-)
Lag length	5	3
R^2	0.37	0.38
GC: F-test	1.8598	2.998
GC: p-value	(0.1)	(0.03)

*Note: The table reports results for the Granger causality test with H0: The money stock MZM does not Granger-cause the public debt. Standard errors of estimates are reported in parentheses. "Lag length" reports the number of lags in the model according to the AIC and BIC. GC: F-test and GC: p-value refer to the F-statistics and the p-value of the Granger causality analysis, respectively. ***, **, and * denote rejection of the H0 at the 1%, 5% and 10% significance level, respectively.*

Table 8: Public debt-to-GDP → MZM velocity, % change from previous year

	1959Q1 - 2019Q4
Debt(-1)	**-0.27*** **
	(0.05)
Debt(-2)	**0.28** **
	(0.09)
Debt(-3)	0.03
	(0.09)
Debt(-4)	-0.04
	(0.07)
Debt(-5)	-0.10
	(0.07)
Debt(-6)	0.09
	(0.05)
Lag length	6
R^2	0.90
GC: F-test	8.9215
GC: p-value	**(3.301e-09)**

*Note: The table reports results for the Granger causality test with H0: The public debt-to-GDP ratio does not Granger-cause money velocity. Data are in percentage changes from previous year. Lags are quarterly, chosen by the BIC. Standard errors of estimates are reported in parentheses. "Lag length" row reports the number of lags in the model according to AIC and BIC. GC: F-test and GC: p-value refer to the F-statistics and to the p-value of the Granger causality analysis respectively. ***, **, and * denote rejection of the H0 at the 1%, 5% and 10% significance level, respectively.*

6.3 Percentage changes from previous year

We perform a set of robustness analyses using growth rates of both quarterly money velocity and public debt-to-GDP with respect to the previous year (four quarters before the observation). The data transformation is the following:

$$x_t = \frac{X_t - X_{t-4}}{X_{t-4}} \tag{9}$$

where x_t represents the growth rate, X_t is the value at time t, and X_{t-4} is the value four quarters before time t. Results are shown in tables 8 and 9. The results confirm a negative and significant coefficient estimate for the first lag of public debt-to-GDP. The estimate of the second lag is significant as well, although it is positive. The p-value of

Table 9: MZM velocity → public debt-to-GDP , % change from previous year

	1959Q1 - 2019Q4
Velocity(-1)	0.21
	(0.17)
Velocity(-2)	-0.24
	(0.29)
Velocity(-3)	0.13
	(0.27)
Velocity(-4)	-0.21
	(0.28)
Velocity(-5)	0.11
	(0.28)
Velocity(-6)	0.07
	(0.18)
Lag length	6
R^2	0.88
GC: F-test	1.2194
GC: p-value	**(0.29)**

*Note: The table reports results for the Granger causality test with H0: Money velocity does not Granger-cause public debt-to-GDP . Data are in percentage changes from previous year. Lags are quarterly, chosen by the BIC. Standard errors of estimates are reported in parentheses. "Lag length" row reports the number of lags in the model according to AIC and BIC. GC: F-test and GC: p-value refer to the F-statistics and to the p-value of the Granger causality analysis respectively. ***, **, and * denote rejection of the H0 at the 1%, 5% and 10% significance level, respectively.*

the F-statistics for the Granger causality test confirm the Granger causality relationship between the velocity and the public debt-to-GDP. Results in table 9 confirm the lack of the reverse Granger causality relationship.

6.4 Inverse velocity of money supply

Tables 10 and 11 show the results of regression and Granger causality for the two time series using the inverse of money velocity, $\frac{M_t}{P_t Y_t}$, and the public debt-to-GDP ratio. The reason for using the inverse of velocity and the debt-to-GDP ratio is to make both variables comparable measures, as they are now expressed as shares of GDP. The results in the two tables confirm the results obtained in the main analysis. The coefficient estimate of the first lag for public debt-to-GDP is significant. As expected, the estimate is now positive and very similar in absolute value to the main analysis.

7 Conclusions

This paper investigates the relationship between the public debt-to-GDP ratio and the money zero maturity (MZM) velocity in the United States. Our findings provide evidence of Granger causality between the public debt-to-GDP ratio and the money velocity, while we cannot establish the reverse Granger causality. Specifically, we observe a negative relationship between the two variables. This result is confirmed at the first lag in all the robustness tests we perform.

From a theoretical standpoint, we use a small model to delve deeper into the transmission mechanisms underlying this one-way relationship. In the case of an expansionary fiscal policy characterised by an increase in government spending, public debt expands, and output increases. As a result, inflation expectations rise, leading to an increase in inflation. Despite the heightened inflation expectations, output increases more than money supply, contributing to an increase in velocity. Conversely, if output increases less than the money supply, the model predicts a decline in the velocity of money supply, consistent with our empirical findings. When considering an expansionary fiscal policy driven by an increase in government transfers, velocity decreases. This is due to a decrease in output driven by the balancing effect that lower government spending exerts on the government budget constraint.

While there is strong evidence of Granger causality between public debt and money velocity, one limit of this paper is worth noting. The empirical analysis only focus on two variables. Both public debt-to-GDP and velocity of money supply can be influenced by a wide range of economic factors, including other type of fiscal policies, monetary policy, and global economic conditions. However, the overall results up to now suggest that the impact of the fiscal sector, including fiscal shocks and public debt-to-GDP may be transmitted to money velocity through the expectations channel. In the next step, we are going to develop a small forward-looking deterministic model to gain a deeper understanding of the mechanisms within this transmission channel.

Table 10: Public debt-to-GDP → inverse MZM velocity

	AIC			BIC		
	1959Q1 - 2019Q4	**1959Q1 - 1995Q4**	**1996Q1 - 2019Q4**	**1959Q1 - 2019Q4**	**1959Q1 - 1995Q4**	**1996Q1 - 2019Q4**
Debt(-1)	**0.26*** **	**0.31*** **	**0.16** **	**0.22*** **	**0.34*** **	**0.16** **
	(0.05)	(0.08)	(0.05)	(0.06)	(0.09)	(0.05)
Debt(-2)	-0.06	0.07	0.08	-	-	-
	(0.06)	(0.09)	(0.08)	(-)	(-)	(-)
Debt(-3)	-0.02	0.10	-	-	-	-
	(0.06)	(0.09)	(-)	(-)	(-)	(-)
Debt(-4)	0.03	0.12	-	-	-	-
	(0.06)	(0.07)	(-)	(-)	(-)	(-)
Debt(-5)	**-0.12***	-	-	-	-	-
	(0.05)	(-)	(-)	(-)	(-)	(-)
Lag length	5	4	2	1	1	2
R^2	0.36	0.35	0.30	0.31	0.32	0.30
GC: F-test	8.4878	11.263	6.0159	30.934	35.721	6.0159
GC: p-value	**(1.073e-07)**	**(1.82e-08)**	**(0.00)**	**(4.441e-08)**	**(6.716e-09)**	**(0.00)**

*Note:The table reports results for Granger causality test with H0: The public debt-to-GDP ratio does not Granger-cause inverse money velocity. Standard errors of estimates are reported in parentheses. "Lag length" reports the number of lags in the model according to the AIC and BIC. GC: F-test and GC: p-value refer to the F-statistics and to the p-value of the Granger causality analysis, respectively. ***, **, and * denote rejection of the H0 at the 1%, 5% and 10% significance level, respectively.*

Table 11: Inverse MZM velocity $\rightarrow$ public debt-to-GDP

	AIC			BIC		
	1959Q1 - 2019Q4	**1959Q1 - 1995Q4**	**1996Q1 - 2019Q4**	**1959Q1 - 2019Q4**	**1959Q1 - 1995Q4**	**1996Q1 - 2019Q4**
Velocity(-1)	0.10	-0.15	-0.19	-0.09	-0.05	-0.19
	(0.11)	(0.12)	(0.32)	(0.10)	(0.10)	(0.32)
Velocity(-2)	0.02	-0.08	-0.24	-	-	-0.24
	(0.17)	(0.17)	(0.33)	(-)	(-)	(0.33)
Velocity(-3)	-0.20	-0.30	-	-	-	-
	(0.15)	(0.19)	(-)	(-)	(-)	(-)
Velocity(-4)	0.15	0.01	-	-	-	-
	(0.12)	(0.10)	(-)	(-)	(-)	(-)
Velocity(-5)	0.04	-	-	-	-	-
	(0.17)	(-)	(-)	(-)	(-)	(-)
Lags	5	4	2	1	1	2
R^2	0.26	0.23	0.30	0.17	0.12	0.30
GC: F-test	1.1436	2.0375	0.92742	0.9033	0.19909	0.92742
GC: p-value	(0.34)	(0.09)	(0.40)	(0.34)	(0.66)	(0.40)

*Note: The table reports results for the Granger causality test with H0: Inverse money velocity does not Granger-cause public debt-to-GDP. Standard errors of estimates are reported in parentheses. "Lag length" reports the number of lags in the model according to the AIC and BIC. GC: F-test and GC: p-value refer to the F-statistics and to the p-value of the Granger causality analysis respectively. ***, **, and * denote rejection of the H0 at the 1%, 5% and 10% significance level, respectively.*

Appendix

This appendix describes the non-linearised and the linearised version of the theoretical model. The notation is the following: upper case variables with a time subscript are variables in levels (e.g. X_t), steady state values are letters without a time subscript (e.g. X), and lower case variables with a hat and a time subscript are linearised variables (e.g. $\hat{x}_t$).

8 Model

Appendix 8.A Household

The household has the following utility function:

$$\max_{C_t, N_t, L_t, B_t} E_t \sum_{t=0}^{\infty} \beta^t \mathcal{U}_t \left(C_t, L_t, N_t\right)$$

with the utility function assuming the form: $\mathcal{U}\left(C, L\right) - \mathcal{V}\left(N\right)$. The budget constraint is:

$$P_t C_t + B_t + M_t = B_{t-1}\left(1 - i_{t-1}\right) + M_{t-1} + W_t N_t + D_t + P_t T_t$$

The maximisation problem is the following:

$$\mathcal{L} = \beta^t \left[\mathcal{U}_t \left(C_t, L_t\right) - \mathcal{V}_t \left(N_t\right)\right] - \lambda_t \left(P_t C_t + B_t + M_t - B_{t-1}\left(1 - i_{t-1}\right) - M_{t-1} - W_t N_t - D_t - P_t T_t\right)$$

The first order conditions are:

$$[C] \quad \lambda_t = \beta^t U_{c,t} \frac{1}{P_t} \tag{10}$$

$$[N] \quad \lambda_t = \beta^t \frac{V_{n,t}}{W_t} \tag{11}$$

$$[B] \quad \beta^t \frac{1}{P_t} \frac{U_{l,t}}{\zeta_t} = \lambda_t - \lambda_{t+1} \tag{12}$$

$$[L] \quad \frac{\lambda_t}{\lambda_{t+1}} = 1 + i_t \tag{13}$$

where $U_{c,t}$, $U_{l,t}$ and $V_{n,t}$ are the first derivatives of $\left[\mathcal{U}_t \left(C_t, L_t\right) - \mathcal{V}_t \left(N_t\right)\right]$ with respect to C_t, L_t and N_t respectively, and λ_t represents the Lagrange multiplier. The Euler equation is obtained by substituting the left hand side of equation (13) with the right hand side of equation (10):

$$U_{c,t} = \beta^t \left(1 + i_t\right) \pi_{t+1}^{-1} U_{c,t+1} \tag{14}$$

where $\pi_t = \frac{P_t}{P_{t-1}}$.

The labour supply equation is derived from equations (10) and (11):

$$\frac{W_t}{P_t} = \frac{V_{n,t}}{U_{c,t}} \tag{15}$$

Finally, the money demand is derived from equations (10), (12) and (13).

From (10):

$$\frac{U_{l,t}}{U_{c,t}} = 1 - \frac{P_t \lambda_{t+1}}{P_t \lambda_t}$$

after cancelling out P_t and substituting $\frac{\lambda_{t+1}}{\lambda_t}$ with the right hand side of equation (13), we obtain:

$$\frac{U_{l,t}}{U_{c,t}} = 1 - \frac{1}{1+i_t} = \frac{i_t}{1+i_t}$$

As in Galí (2020) we define $\frac{U_{l,t}}{U_{c,t}} = h\left(\frac{L_t}{C_t}\right)$. The money demand can be written as:

$$\frac{U_{l,t}}{U_{c,t}} = h\left(\frac{L_t}{C_t}\right) = \frac{i_t}{1+i_t} \tag{16}$$

Appendix 8.B Firms

8.B.1 Final good firms

The final good firms produce their goods with the following technology:

$$Y_t = \left(\int_0^1 Y_{it}^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$$

where Y_{it} are intermediate goods that are packed into the final good. Firms maximise profits subject to:

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\epsilon_t} Y_t$$

8.B.2 Intermediate firms

Intermediate goods are produced with technology:

$$Y_{it} = N_{it}^{1-\alpha}$$

The nominal marginal cost MC_t^n is given by:

$$\begin{aligned} MC_t^n &= \frac{W_t}{MPN_t} \\ &= \frac{W_t}{(1-\alpha)N_t^{-\alpha}} \\ &= W_t(1-\alpha)^{-1}N_t^{\alpha} \end{aligned}$$

with $MPN_t = \frac{\partial Y_t}{\partial N_t}$ being the marginal product of labour.
Intermediate firms adjust their prices according to the Calvo price setting, as outlined in Calvo (1983). This implies that a fraction of firms, denoted as θ, are unable to change their prices over time. On the other hand, a fraction of $1-\theta$ firms have the flexibility to adjust their price over time, and they set P_t^*. P_t^* is the same price for all the firms that are adjusting. The firms that have the ability to change their prices take into account the potential impact on future profits when deciding on a price adjustment today.

The aggregate price dynamics is

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon}$$

and the linearised version of it, doing a first order Taylor expansion:

$$\pi_t = (1-\theta)\left(p_t^* - p_{t-1}\right) \tag{17}$$

The firms' optimising problem is:

$$\max_{P_t^*}\left\{\sum_{k=0}^{\infty}\theta^k E_t\left[\left(\beta^k\left(\frac{C_{t+k}}{C_t}\right)^{-\sigma}\frac{P_t}{P_{t+k}}\right)\left(P_t^* Y_{it+k|t} - TC_{it+k|t}^n\left(Y_{it+k|t}\right)\right)\right]\right\} \tag{18}$$

under the following set of demand constraints:

$$Y_{it+k|t} = \left(\frac{P_t}{P_{t+k}}\right)^{-\epsilon} Y_{t+k} \tag{19}$$

where P_t^* is the price that maximises present value of profits while having that price and it is set by the firms. $Y_{it+k|t}$ is the output produced while having that price and $MC_{it+k|t}^n$ is the marginal cost a firm faces given that price. $\beta^k\left(\frac{C_{t+k}}{C_t}\right)^{-\sigma}\frac{P_t}{P_{t+k}}$ is the discount factor derived from the Euler equation. The first order condition for price P_t^* is

$$E_t\sum_{k=0}^{\infty}\beta^k\theta^k Q_{t,t+k}Y_{it+k|t}\left(P_t^* - \mu_{t+k}\, MC_{t+k|t}^n\right) = 0 \tag{20}$$

where $Q_{t,t+k} = \beta^k\left(\frac{C_{t+k}}{C_t}\right)^{-\sigma}\frac{P_t}{P_{t+k}}$ is the nominal discount factor for firms and the discount factor for the households.

Appendix 8.C Steady state equations

First, the price mark-up has to be derived to obtain the first steady state equation. To do so, the MPN_t^n marginal productivity of labour is defined as:

$$MPN_t^n = \frac{\partial Y_t}{\partial N_t} = (1-\alpha)\,N_t^{-\alpha} \tag{21}$$

As in Galí (2015), the nominal marginal cost using labour is W_t. The nominal marginal gain of firms by using labour is the income increase, that is the price times the marginal increase in production by adding one unit more of labour. Thus, the real marginal cost is the nominal cost relative to the nominal gain:

$$MC_t^r = \frac{W_t}{P_t \, MPN_t^n} \tag{22}$$

Substituting for MPN_t as in equation (21) we obtain:

$$MC_t^r = \frac{W_t}{P_t \, (1-\alpha) \, N_t^{-\alpha}} \tag{23}$$

As the firms' mark up is equal to the inverse of the real marginal cost:

$$\mu_t = \frac{(1-\alpha) \, P_t}{W_t N_t^{\alpha}} \tag{24}$$

Where μ_t is the price mark-up.

In steady state, the mark-up is equal to the desired mark-up:

$$\mu = \frac{(1-\alpha) \, P}{W N^{\alpha}} = \frac{\epsilon}{\epsilon - 1} \tag{25}$$

where A is normalised to 1. Denoting $\frac{\epsilon}{\epsilon-1} = \mathcal{M}$, from (25) it follows that:

$$(1-\alpha) \, P = \mathcal{M} W N^{\alpha} \tag{26}$$

and considering equation (15) evaluated at steady state,

$$\frac{W}{P} = \frac{V_n}{U_c} \tag{27}$$

equation (26) becomes:

$$(1-\alpha) = \mathcal{M} \frac{W}{P} N^{\alpha} \tag{28}$$

$$(1-\alpha) = \mathcal{M} \frac{V_n}{U_c} N^{\alpha} \tag{29}$$

$$(1-\alpha) \, U_c = \mathcal{M} V_n N^{\alpha} \tag{30}$$

which is equivalent to writing:

$$(1-\alpha) \; U_c \left(N^{1-\alpha}, L \right) = \mathcal{M} \, V_n \, (N) \, N^{\alpha} \tag{31}$$

The second equation describing the steady state is obtained from the money demand:

$$h \left(\frac{L}{C} \right) = \frac{i}{1+i} \tag{32}$$

where, from the definition of β and the Euler equation evaluated at steady state, $i = \rho$. Therefore, equation (32) can be rewritten as:

$$h \left(\frac{L}{N^{1-\alpha}} \right) = \frac{\rho}{1+\rho} \tag{33}$$

Appendix 8.D Linearised model

8.D.1 Economic identity

$$\hat{y}_t = \hat{c}_t + \hat{g}_t \tag{34}$$

8.D.2 Euler equation

$$\hat{\xi}_t = \hat{\xi}_{t+1} + \hat{i}_t - \pi_{t+1} \tag{35}$$

8.D.3 Non-separable household utility function

This equation describes the two linearised components of the utility function $U(C, L)$.

$$\begin{aligned} \hat{\xi}_t &= ln\left(\frac{U_{c,t}}{U_c}\right) \\ &= \hat{c}_t C \frac{U_{cc}}{U_c} + \hat{l}_t L \frac{U_{cl}}{U_c} \\ &= -\sigma \hat{c}_t + \nu \hat{l}_t \end{aligned} \tag{36}$$

where $U_{c,t} = U(C_t, L_t)$, $\sigma \equiv -C\frac{U_{cc}}{U_c}$ and $\nu \equiv L\frac{U_{cl}}{U_c}$

We follow Galí (2020) and we assume $\nu = 0$, which implies separability in the consumer's utility function between consumption and real balances.

8.D.4 Price mark-up

$$\hat{\mu}_t = \hat{\xi}_t - \hat{y}_t\left(\frac{\varphi + \alpha}{1 - \alpha}\right) \tag{37}$$

8.D.5 Money demand

$$\hat{l}_t = \hat{c}_t - \eta \hat{i}_t$$

where:

$$\eta = \frac{\epsilon_{l,c}}{\rho} \text{ and } \epsilon_{l,c} = -\frac{1}{h'}\frac{\rho}{1+\rho}V = \frac{1}{\sigma_l + \nu}$$

8.D.6 New Keynesian Phillips Curve

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \widehat{mc}_t^r \tag{38}$$

where

$$\lambda = \frac{(1-\theta)(1-\theta\beta)}{\theta} \tag{39}$$

8.D.7 Definition of money growth

$$\Delta \hat{m}_t = \hat{l}_t - \hat{l}_{t-1} + \pi_t \tag{40}$$

where $\hat{m}_t - \hat{m}_{t-1} = ln\left(\frac{M_t}{M_{t-1}}\right) = \Delta \hat{m}_t$ and $\hat{p}_t - \hat{p}_{t-1} = ln\left(\frac{P_t}{P_{t-1}}\right) = \pi_t$.

8.D.8 Government budget constraint

$$\hat{b}_t = (1+\rho)\,\hat{b}_{t-1} + \bar{b}(1+\rho)(\hat{i}_{t-1} + \hat{\pi}_t) + \hat{g}_t + \hat{t}_t - \frac{\bar{m}}{\bar{y}}(\hat{l}_t - \hat{l}_{t-1} + \hat{\pi}_t) \tag{41}$$

8.D.9 Fiscal rules

We follow Leeper et al. (2010) in setting the fiscal rules, with the exception that in our baseline analysis, we use fiscal rules based on debt only. The fiscal rule and the AR(1) transfer shock process for transfers are:

$$\hat{t}_t = -\psi_{bt}\,\hat{b}_{t-1} + \hat{t}^*_t \tag{42}$$

$$\hat{t}^*_t = \delta_t\,\hat{t}^*_{t-1} + \epsilon_t \tag{43}$$

and for government spending:

$$\hat{g}_t = -\psi_{bg}\,\hat{b}_{t-1} + \hat{g}^*_t \tag{44}$$

$$\hat{g}^*_t = \delta_g\,\hat{g}^*_{t-1} + \epsilon_g \tag{45}$$

8.D.10 Central bank

The Taylor rule reads:

$$i_t = \phi_\pi \pi_t + \phi_y y_t \tag{46}$$

where ϕ_π and ϕ_y represent the weight on the interest rate of inflation and output respectively.

8.D.11 Velocity identity

Finally, the linearised velocity equation, transformed in real terms, is the following:

$$\hat{v}_t \equiv \hat{y}_t - \hat{l}_t \tag{47}$$

9 Data

The data used in this study were obtained from the FRED database of the Federal Reserve Bank of St. Louis. The nominal public debt is represented by the Market Value of Marketable Treasury Debt (MVMTD027MNFRBDAL) expressed in billions of dollars. Gross Domestic Product (GDP) is used as the measure of the economy's output, also in billions of dollars. The MZM Money Stock (MZM) represents the money supply.

To analyse the data, we calculated the growth rates of each variable with respect to the previous quarter.

$$x_t = \frac{X_t - X_{t-1}}{X_{t-1}} \tag{48}$$

where $x_t = \frac{MZM}{GDP}, \frac{MVMTD027MNFRBDAL}{GDP}$

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